Amplification factors in shock-turbulence interactions: Effect of shock thickness

Diego A. Donzis[a)]
Department of Aerospace Engineering, Texas A&M University, Texas 77843-3141, USA

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Amplification factors of streamwise velocity are investigated in canonical shock-turbulence interactions. The ratio of laminar shock thickness to the Kolmogorov length scale is suggested as the appropriate parameter to understand data from simulations and experiments. The different regimes of the interaction suggested in the literature can also be understood in terms of this parameter. © *2012 American Institute of Physics*. [doi:10.1063/1.3676449]

The interaction of turbulence with shock waves is an important phenomenon in a variety of contexts including supernovae explosions, supersonic aerodynamics and propulsion, among others. To understand the complexities of such flows, substantial efforts have been devoted to the simplest case of isotropic turbulence interacting with a normal shock. A widely used measure of the effect of a shock wave on turbulence is the so-called amplification factors which are defined as the ratio of suitably defined averages (typically plane averages for simulations or time averages for experiments) of quantities of interest after and before the shock. Our interest here is restricted to amplification factors of velocity components normal to the shock, $G \equiv u_d'2/u_u'2$ where prime stands for root-mean-square quantities and subscripts u and d stand for upstream and downstream of the shock, respectively. Ribner[1,2] and Moore[3] assumed the shock to be a discontinuity and the incoming flow to be a superposition of simple waves and obtained analytical expressions for a variety of amplification factors. Since this theory is based on the linearized Euler equations, it is usually referred to as the Linear Interaction Analysis, or LIA. A number of other theoretical approaches have been proposed to predict the effect of a shock on velocity, vorticity, or other quantities.[4–10] However, LIA is currently the most widely used approach to try to understand experimental and numerical data, in part, because of its relative success over other approaches and its ability to provide analytical results for a range of quantities of interest.

Evidence from experiments and simulations, however, has consistently shown that the interaction depends on characteristics of the incoming flow not accounted for in LIA, such as velocity and length scales.[11] These dependencies, though, are not understood even in a general qualitative manner. It has been further suggested that the interaction can be in either the "wrinkled" or "broken" regimes[12–14] depending on whether the shock preserves its identity as a well defined steep gradient or its structure is fundamentally modified. The theoretical approaches mentioned above cannot account for this behavior. In particular, the relation between the regime of the interaction and the scaling of amplification factors has not been investigated in any detail. Since the regime is based on details of the structure of the shock, it is natural to inquire about the role of the shock thickness in the scaling of the interaction.

In view of this situation, it seems desirable to examine the most recent data available on amplification factors, investigate the extent to which assumptions needed for theories are satisfied, and establish how G scales with governing parameters, especially the shock thickness.

Elemental dimensional analysis on the problem of a stationary shock interacting with isotropic turbulence yields

$$G = f(M_t, R_\lambda, \Delta M), \tag{1}$$

[a)]Electronic mail: donzis@tamu.edu.

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where $M_t \equiv \sqrt{3}u'/c$ is the turbulent Mach number,[15] c the mean speed of sound, $R_\lambda \equiv u'\lambda\rho/\mu$ is the Taylor Reynolds number, λ is the Taylor microscale, ρ and μ are the mean density and viscosity and f is an unknown universal function. The parameter $\Delta M \equiv M - 1$ is chosen over M (the mean Mach number) for convenience.

To meaningfully compare different flows several conditions have to be met. First, flows have to be geometrically similar. In our case, we focus on a stationary shock interacting with isotropic turbulence consisting mainly of vortical fluctuations.[16] This leaves out some experiments[17] where the shock moves through spatially and temporally decaying turbulence and some data from simulations where entropy or acoustic fluctuations are artificially introduced at the inlet. Second, the governing parameters should be obtained at the same location. A natural choice is to use values immediately upstream of the shock wave, a convention used throughout the literature and hereafter (unless noted). This location is easily identified in simulations as the streamwise rms velocity reaches a well defined minimum. Third, to properly define G, a location downstream of the shock has to be identified to measure $u_d'^2$. In simulations it is common to use the location where u'^2 peaks after the shock.[18] This extremum has been consistently found in simulations, though has not been observed in experiments, presumably because of the difficulties in measuring close to the shock. This point is also critical to compare with theories. LIA provides results in the so-called near field, where quantities undergo variations as a function of the distance from the shock, and the far field which represent the asymptotic value far away from the shock. Although amplification factors are typically compared against far-field predictions, due to the viscous decay after the shock in simulations, it is not clear where the asymptotic value can be found. Still, as commonly adopted in practice, we use the peak mentioned above to compare with LIA (though others[14] extrapolate u' to the nominal shock location.) Fourth, either Reynolds or Favre averages have to be used to compute G. Given the already scarce data available, it is unfortunate that experimental and numerical studies have used Reynolds and Favre averages, respectively. The data presented here comes mostly from direct numerical simulations (DNS) using Favre averages.

The studies we consider are listed in Table I and include shock-resolving[13,19] and shock-capturing[14,20–22] simulations as well as an experimental study[23] which satisfy as closely as possible the conditions above (one difference is the use of Reynolds averages). Fig. 1 shows the collection of amplification factors from these studies as a function of ΔM. This representation (often using simply M for the abscissa) is based on LIA which is formally valid only for $M_t \to 0$ and $R_\lambda \to \infty$ in which limit, the dependence on these parameters is assumed to vanish,

$$G = f_{LIA}(\Delta M). \tag{2}$$

TABLE I. Sources: For all simulations the inflow is statistically isotropic and is convected into the domain using Taylor's hypothesis. All values correspond to conditions right before the shock, except for those in parenthesis which are at the inlet of the domain. SC and SR stand for shock-capturing (open symbols) and shock-resolving (closed symbols) simulations respectively.

Source	M_t	R_λ	M	Method	Inflow	Symbol
Lee, Lele, and Moin[13]	0.0567–0.11	12–20	1.05–1.20	SR	"Synthetic" with model spectrum, no thermodynamic fluctuations	▶
Hannappel and Friedrich[20,a]	(0.17)[b]	(6.67)	(2.0)	SC	"Synthetic" with model spectrum	◇
Lee, Lele, and Moin[21]	0.09–0.11	15.7–19.7	1.5–3.0	SC	Decaying compressible turbulence	□
Mahesh, Lele, and Moin[22]	(0.14)	(19.1)	(1.3)	SC	Decaying compressible turbulence	Δ
Jamme *et al.*[19]	0.173	5–6	1.20–1.50	SR	Decaying compressible turbulence	●
Larsson and Lele[14,c]	0.16–0.38	40	1.3–6.0	SC	"Blended" decaying compressible	▽
Barre, Alem, and Bonnet[23]	0.011	15	3	Experiment		◀

[a]Amplification factors taken from Jamme *et al.*[19]
[b]Numbers in this reference are slightly different due to different definitions.
[c]Amplifications factors here are different than in this reference since values were extrapolated to the shock location from far-field conditions. Values presented here correspond to the location of the peak as described in the text and were provided by the authors.

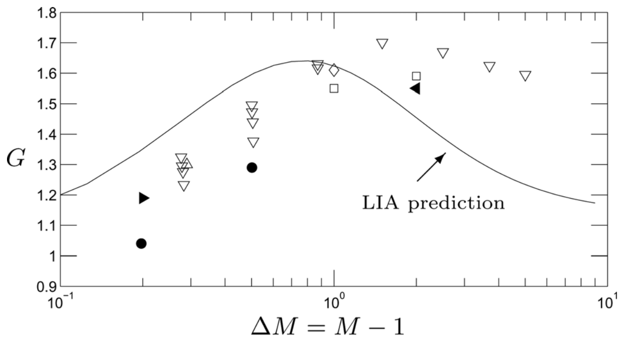

FIG. 1. Amplification factors from references in Table I.

This type of reduction in the number of similarity parameters in specific limits is usually referred to[24] as complete similarity (or similarity of the first kind), in our case, in R_λ and M_t.

Clear discrepancies between data and theory are seen which do not appear to decrease as ΔM increases. Careful investigation of the data reveals systematic trends at fixed ΔM. For example, Reynolds and turbulent Mach numbers effects are observed at $\Delta M \approx 0.5$ where the triangles correspond to simulations at $R_\lambda \approx 40$ with M_t varying from 0.155 (top) to 0.375 (bottom) while the closed circle corresponds to $R_\lambda \approx 5.5$ and $M_t \approx 0.133$. Taken together, the data suggest both a systematic decrease of G with M_t and an increase with R_λ.

Because a number of theoretical approaches, including LIA, assume the shock to be a discontinuity, their validity rests on the comparison of the shock thickness to other relevant scales, in particular the smallest dynamically active scale in the incoming turbulence—the Kolmogorov scale $\eta \equiv (\nu^3/\langle\epsilon\rangle)^{1/4}$ where $\langle\epsilon\rangle$ is the average dissipation rate (angular brackets represent a suitable average). Since, for moderate Mach numbers, the laminar shock thickness scales[25] as $\delta_l \approx (\mu/\rho c)\Delta M^{-1}$, one can easily show that

$$\delta_l/\eta \approx K, \tag{3}$$

where $K \equiv M_t/(R_\lambda^{1/2}\Delta M)$ for which we have use the well-known relation $\langle\epsilon\rangle \sim \nu u'^2/\lambda^2$. The precise value this ratio should assume in order to make theories applicable, cannot be obtained from dimensional considerations alone and should be compared against data.

Furthermore, from a physical perspective one can argue that when turbulence interacts with a rapid mean deceleration such as a shock, a relevant non-dimensional parameter governing the interaction would be the ratio of characteristic scales of the turbulence to those of the shock. Such a similarity parameter may indeed be K.

This parameter also seems to be a natural choice to demarcate the different regimes of the interaction. Because K is the ratio of the laminar shock thickness to the Kolmogorov length scale, for relatively small values of K, the shock is expected to be subjected to a locally uniform velocity field and will therefore behave as a sharp front corresponding to the laminar solution—that is the "wrinkled" regime. On the other hand, for relatively large K, the turbulence can greatly disturb the shock, creating multiple compression peaks, or even smooth compressions—that is, the "broken" regime.

We are thus interested in investigating the scaling,

$$G = f^*(K). \tag{4}$$

From a similarity scaling perspective,[24,26] it is interesting to highlight the difference between Eqs. (2) and (4). While the number of similarity parameters in both have been reduced to one, Eq. (2) represents a complete similarity solution where two parameters are neglected in the limit of being very large or very small, whereas Eq. (4) represents incomplete similarity (or of the second kind) in that the number of parameters is reduced by combining powers of the original similarity parameters: dependencies on viscosity, turbulence intensity, and length scales are retained.

To test Eq. (4) against the data, in Fig. 2 we show the collection of amplification factors as a function of K. A more universal behavior emerges as results from different sources and at different conditions appear to follow a single curve; unlike Fig. 1 no systematic trends are apparent for fixed K which supports K as a more appropriate non-dimensional parameter to characterize the interaction, at least, under the conditions in simulations and experiments. Overall, the amplification factor takes larger values at low K (around 1.6) and lower values at high K (close to unity at the highest K available).

To try to understand the behavior of G we first consider the limit $\Delta M \to \infty$ (i.e., $K \to 0$). Under all the conditions necessary to make LIA a valid approximation, it is possible to obtain analytically an asymptotic amplification of $G \approx 1.14$ for $\gamma = 1.4$.[9] An asymptotic amplification is not inconsistent with the data in Fig. 2, though the limiting value (horizontal dashed line) appears to be about 40% larger than LIA prediction. We note, however, that besides $K \to 0$, LIA requires *simultaneously* $R_\lambda \to \infty$ and $M_t \to 0$. Due to the significant viscous decay observed in the simulations, though, it is unclear to what extent an inviscid approximation is justified.

At higher values of K, the amplification factor decreases with K suggesting that as the disparity between turbulence scales and the shock thickness decreases, the overall effect of the shock on the incoming flow weakens. This is qualitatively consistent with theoretical[8,27] and numerical results[14] which suggest that jumps under turbulent conditions are weaker than in laminar flows. In this range, a power law of the form $G \approx s_1 K^{-s_2}$ is found to represent the data well with $s_1 \approx 0.75$ and $s_2 \approx 1/4$ (Fig. 2).

In the other limit $\Delta M \to 0$, and under an inviscid assumption, the shock becomes a weak Mach wave and the velocity jump vanishes, i.e., $G \to 1$. The available data in the literature do not appear to achieve high enough K to test such an asymptotic state (Fig. 2). However, we are interested in finite Reynolds numbers where δ_l is finite and the turbulence undergoes a viscous decay. If one could let $\Delta M \to 0$ such that δ_l remains finite but the effect on turbulence (at least on large areas across the shock) is negligible, then the amplification factor will simply reflect the decay of u'^2 over a distance δ_l. Using the classical estimates $du'^2/dt \approx -\langle\epsilon\rangle \approx -Au'^3/L$ and $u'^2 L^3 =$ constant (as a result of the constancy of Saffman integral[28]) and integrating we obtain[29] $G \approx [1 + C(u_0/L_0)t]^\sigma$ (where $C = 5A/6$, $\sigma = -6/5$, and $A \approx 0.4$). Using Taylor hypothesis ($t = x/U$), the decay over δ_l is $G = [1 + C(u_u/L_u)(\delta_l/U)]^\sigma = [1 + C(M_t/M)KR_\lambda^{-3/2}]^\sigma$ which, for M not too close to unity or fixed M, may be approximated by $G \approx (1 + CK^2/R_\lambda)^\sigma$, also shown in Fig. 2 at high K for $R_\lambda \approx 5$, 10, and 20 (dashed-dotted lines). This result suggests that at high K a R_λ-dependence reappears. For fixed Reynolds number, the asymptotic state would scale as $G \sim K^{2\sigma}$. It is not clear whether such an asymptotic state with a steady shock is realizable in practice, though, as this regime may indeed correspond to "destructive interactions"[30] in which vigorous turbulence interacts with a very weak shock. Clearly, high-fidelity (shock-resolving) data are necessary to establish this regime with any certainty.

It is important to recognize the difficulties in DNS due to the conflicting requirements to resolve broadband turbulence and capture shocks which make numerical details have a significant effect on the solution.[31] For example, at low K, while Ref. 21 used ENO for inviscid fluxes only in the streamwise direction and close to the shock and sixth-order compact schemes for the other terms and in other regions, Ref. 14 used WENO in all directions when a sensor detects steep gradients, otherwise, an explicit sixth-order central scheme is used. One wonders if the numerical details of different simulations (in particular if grid-convergence tests have not been performed[14])

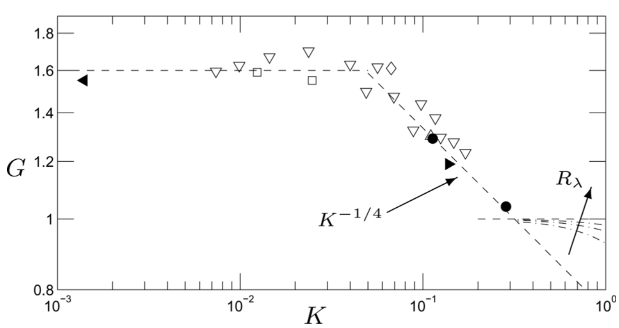

FIG. 2 Amplification factors from Fig. 1 according to Eq. (4).

contribute to some of the scatter across studies and whether shock-resolving simulations (solid symbols) would provide more consistent results at lower K. In addition, since one cannot rule out an influence of the details of the turbulence at the inlet (see Table I) as well as some statistical variability, the good collapse in Fig. 2 appears to be quite satisfactory.

As noted, K may also be indicative of the different regimes in the interaction. Ref. 13 suggested $M_t^2/(M^2-1)$ as the parameter controlling the boundary between regimes. Ref. 14, however, suggests that data does not support this as the appropriate parameter though it provides some reasonable guidance with a transition around $M_t^2/(M^2-1) \approx 0.06$ which corresponds to $K \approx 0.1$ at the conditions of the simulations. This is in fact close to the midpoint between asymptotic states in Fig. 2. Thus, the transition between asymptotes in the figure appears to correspond to the transition between "wrinkled" and "broken" regimes. Data in Ref. 13 are also consistent with this finding: simulations at $K \approx 0.07$ (estimated from the conditions stated in that reference) had relatively well-defined fronts while at $K \approx 0.48$ multiple compression waves were seen along individual streamlines.

We note, however, that the definition of "wrinkled" and "broken" regimes in the literature is typically based on a *qualitative* assessment of contour levels or instantaneous profiles of pressure or density. The transition in Fig. 2, on the other hand, is relatively smooth and no particular value of K can be chosen unambiguously to delimit "wrinkled" and "broken" regimes. Therefore, using the value of K to characterize the interaction seems to be more appropriate: lower K would correspond to better defined sharp fronts across the entire shock surface while higher K would correspond to increasingly distorted fronts and where increasingly large areas may present multiple or smooth compressions.

We have investigated the extent to which $K = \delta_l/\eta$ can characterize amplification factors from simulations and experiments at finite Reynolds and Mach numbers. The main result shown in Fig. 2 shows that K can, at least to first order, collapse available data into a universal curve. This result could also be useful to design simulations and experiments depending on what regime is of interest. In particular, shock-resolving simulations at a range of values of K especially in asymptotic regimes will be highly desirable.

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